Geometric Classification of Topological Quantum Phases

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Abstract

On the basis of the principle that topological quantum phases arise from the scattering around space-time defects in higher dimensional unification, a geometric model is presented that associates with each quantum phase an element of a transformation group.

1 Introduction

Within a physical theory there are often effects or phenomena studied in idealized form in order to gain insight into the peculiarities of the theory. In Quantum Mechanics one such effect is the appearance of topological quantum phases in the wave functions of particles moving freely in multiply connected space-times the prototype of this effect being the Aharonov-Bohm (AB) effect [1], the appearance of a phase factor in the wave function of an electron which moves around a magnetic flux line. A similar effect is the Aharonov-Casher (AC) effect [2] which is obtained from the AB effect by replacing the flux line and the electron by a charged line and a neutral particle with magnetic moment, respectively. There have also been studied analogous effects in gravitation [3][4][5][6][7] and in non-Abelian gauge theory [8]. Moreover, there have been considered quantum phases associated with higher multipole moments of charges [9].

It seems that topological quantum phases appear generically in theories that allow a geometric, gauge theoretic formulation. Up to now, however, a unified description is lacking.

In this letter, a combined formulation of topological quantum phases by means of a general model is proposed. This model provides a classification and — to a certain extent — also a prediction of topological quantum phases. The basic idea of this letter is the view of the "flux line" that generates the multiple connectedness of space-time as a topological defect similar to a crystal line defect. We use defects in higher dimensional space-times in order to describe internal gauge interactions in the framework of higher dimensional unification. The model is formulated as a gauge theory generalizing gauge theory models of gravitation in that curvatures of higher order are introduced.

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The letter is organized as follows: In section 2, we begin — as motivation and illustration of the model — with a comparison of AB and AC effects in electromagnetism and gravitation. In section 3, we formulate the model in a general framework and give some examples. Section 4 contains a summary and some comments.

2 AB and AC Effect in Electromagnetism and Gravitation

In the electromagnetic AB effect, the wave function of a charge q experiences a phase change when the charge moves around a magnetic flux line. The phase factor is given by

$$\Lambda_{em}^{AB} = \exp\left(\frac{i}{\hbar}\oint qA_\mu dx^\mu\right) = \exp\left(\frac{i}{\hbar}q\phi\right), \tag{1}$$

where A_μ ($\mu = 0,\ldots,3$) is the 4-vector potential of the flux line with flux ϕ and the integration is along an arbitrary curve surrounding the flux line. The field strength $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$ is singular on the flux line.

The gravitational AB effect arises when the wave function of a massive particle encircles a spinning cosmic string. Such a string is conveniently described by a singularity of torsion [10]. The phase factor is

$$\Lambda_{gr}^{AB} = \exp\left(\frac{i}{\hbar}\oint p_a e_\mu^a dx^\mu\right) = \exp\left(\frac{i}{\hbar}8\pi GmS\right). \tag{2}$$

Here, p_a ($a = 0,\ldots,3$) is the momentum of the particle with mass m, e_μ^a represents the vierbein of the string geometry with spin S per unit length, and G is the gravitational constant. e_μ^a plays the role of a potential for the torsion $T_{\mu\nu}^a = 2\partial_{[\mu}e_{\nu]}^a$. Since there is no curvature present, we can use a teleparallel formulation of gravitation and choose a gauge in which the Lorentz connection vanishes identically.

Comparing the two phases (1) and (2) we see that they have a similar form. Indeed, we can combine these phases into a single phase if we use a unification of gravitation and electromagnetism through a 5-dimensional teleparallel gravitation akin to the Kaluza-Klein model [11]. However, we do not employ a 5-dimensional metric. On the manifold $M_4 \times S^1$ where M_4 is space-time we introduce a fünfbein $E_M^A(A, M = 0,\ldots,3,5)$ with components

$$E_\mu^a = e_\mu^a, \quad E_5^a = 0, \quad E_\mu^5 = A_\mu, \quad E_5^5 = 1.$$

In this case, the 5-th component $T_{\mu\nu}^5$ of the torsion tensor is the field strength $F_{\mu\nu}$. Since the charge q represents the 5-th component of the 5-momentum p_A, the unified AB phase is $\hbar^{-1}\oint\left(p_A E_\mu^A dx^\mu\right)$.

We now turn to the AC effect. Instead of the usual electromagnetic AC effect we consider its dual effect, that is, the scattering of an electric dipole moment from a straight line of magnetic monopoles the dipole being polarized along the line [12]. The phase factor reads

$$\Lambda_{em}^{AC} = \exp\left(\frac{i}{\hbar}\oint(\mathbf{B}\times\mathbf{d})\cdot d\mathbf{r}\right) = \exp\left(\frac{i}{\hbar}d_z\lambda\right), \tag{3}$$

where $\mathbf{d}$ is the dipole moment, d_z its z-component, and $\mathbf{B}$ the radial magnetic field of the monopole line which lies on the z-axis with magnetic charge λ per unit length.

The counterpart to this effect in gravitation consists in the scattering of a spinning particle from a massive cosmic string with the spin polarized along the string. The phase factor is

$$\Lambda_{gr}^{AC} = \exp\left(\frac{i}{2\hbar}\oint J_{ab}\omega_\mu^{ab}dx^\mu\right) = \exp\left(\frac{i}{\hbar}8\pi Gs_z M\right). \tag{4}$$

Here, J_{ab} is the spin of the particle, s_z its z-component, and ω_μ^{ab} the Lorentz connection of the string with mass M per unit length on the z-axis.

While these two AC effects are physically analogous their mathematical formulations are fundamentally different: The gravitational phase factor (4) represents the holonomy of a locally flat Lorentz connection. The electromagnetic phase factor (3), however, cannot be viewed as the holonomy of a locally flat connection. This discrepancy is resolved in the following way:

The gravitational AC effect was formulated by means of a linear connection the cosmic string representing a curvature singularity. We consider instead a teleparallel formulation of gravitation in which the curvature is set to zero but a nonvanishing torsion is allowed. The interference of neutral spin-$\frac{1}{2}$ particles in gravitational fields with torsion was investigated in [13]. In the teleparallel case, the phase operator is

$$\Lambda_{gr} = \mathcal{P}\exp\left(-\frac{i}{2\hbar}\oint \hat{S}^{ab}e_a^\mu e_b^\nu T_{\rho\mu\nu}dx^\rho\right), \tag{5}$$

where $\hat{S}^{ab}$ is the spin operator, $T^\rho{}_{\mu\nu} = e_a^\rho T^a{}_{\mu\nu}$ the torsion tensor, and e_a^μ the inverse of e_μ^a. Roman indices are raised and lowered with the Minkowski metric $\eta_{ab} = \text{diag}(-1,1,1,1)$ or its inverse, greek indices with the space-time metric defined by $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$. A solution for a massive straight cosmic string in teleparallel gravitation is given by the vierbein

$$e^0 = dt, \quad e^1 = r^{-4GM}dx, \quad e^2 = r^{-4GM}dy, \quad e^3 = dz, \tag{6}$$

and the Lorentz connection $\omega_\mu^{ab} = 0$ ($r^2 = x^2 + y^2$). Equation (6) is equivalent to the solution of a massive particle in (2+1)-dimensional teleparallel gravitation [14]. Inserting the solution (6) into the phase operator (5), we recover the phase factor (4) of the gravitational AC effect where $\hat{S}^{ab}$ has the only nonvanishing eigenvalue $S^{12} = s_z$. Returning to the electromagnetic AC effect we can write the phase factor (3) covariantly as

$$\Lambda_{em}^{AC} = \exp\left(\frac{i}{\hbar}\oint d^\mu F_{\mu\nu}dx^\nu\right) = \exp\left(\frac{i}{\hbar}\oint d^a e_a^\mu T_{\mu\nu}^5 dx^\nu\right), \tag{7}$$

where we have finally written the field strength $F_{\mu\nu}$ as the 5-th component of the torsion tensor in the 5-dimensional unification introduced above. Moreover, we have referred the dipole moment to the vierbein e_μ^a. The formal similarity of expression (7) with expression (5) — together with the interpretation of the field strength as torsion in a 5-dimensional unification — suggests that Maxwell's formulation of

electromagnetism represents a teleparallel formulation provided the view of a higher dimensional unification is adopted. This is the reason why the electromagnetic ACeffect is formulated differently from the gravitational one. Since the gravitational AC effect admits a formulation involving only curvature the same should be possible for the electromagnetic effect. This is indeed the case as the following considerations show:

Both a massive and a spinning straight cosmic string represent space-time defects [15]. These defects can be thought of as being created through global cutting and pasting processes (Volterra process) in which space-time points are identified by means of symmetry transformations. The geometries of defects can be described by locally flat connections associated with the groups of symmetry transformations.

A spinning cosmic string along the spatial z-axis is a space-time defect in the (z, t)-plane. It can be generated by cutting space-time along a hypersurface bounded by the (z, t)-plane and identifying the borders after a mutual translation in time direction. In the terminology of the theory of crystal defects, this defect is a screw dislocation with Burgers vector in time direction.

From a 5-dimensional point of view, also a magnetic flux line can be considered as a topological defect. In this case, 5-dimensional space-time is cut along a hypersurface bounded by the flux line and the cut surfaces are identified after a constant mutual $U(1)$-transformation of the internal S^1-space has been performed. If this transformation is viewed as a translation, the flux line corresponds to a screw dislocation.

A massive straight cosmic string has its counterpart in crystal physics in a wedge disclination. Its geometryis generated by identifying points related by a rotation around the string. Equivalently, it can be thought of as being created through removal of a wedge from space. The geometry is described in a natural way by a linear connection which has a curvature singularity on the string. In the teleparallel formulation of the massive cosmic string, the defect generating rotation is considered as being local translations spread out over space. The resulting geometry can be illustrated by a continuous distribution of dislocations parallel to the string these being, however, edge dislocations which are created in the Volterra process through translations perpendicular to the defect line.

In the comparison of the gravitational AC effect with the electromagnetic one, we have seen above that both effects have a similar description in a teleparallel formulation. This suggests that also a line of magnetic charges represents a topological defect, being associated — like a wedge disclination — with a linear transformation. The magnetic field of a line of monopoles corresponds to a continuous distribution of radially outgoing flux lines which we have interpreted as screw dislocations. In the same way as the rotation that generates a wedge disclination can be considered as local translations, the local $U(1)$-transformations that generate the magnetic field of the monopole line can be regarded as a single linear transformation. In fact, this transformation is an internal $U(1)$-transformation linear in the z-coordinate if the monopole line is directed along the z-axis. A linear connection which describes a line of monopoles as a curvature singularity is associated to this linear transformation. The electromagnetic AC phase can be looked upon as the holonomy of this

connection.

It should be remarked that also a line of electric charges can be interpreted as a disclination if the 4-vector potential of the dual field strength is used as will be described below.

To summarize this section, we have shown that the AB and AC effects in electromagnetism and gravitation can be regarded as being associated with topological space-time defects. In the following section we will generalize this result.

3 Classification Model

Motivated by the considerations in the previous section we will formulate in this section a mathematical model for the description of topological quantum phases based on the principle that the "flux lines" in the effects represent topological defects.

To this end we consider defects on a $(4+D)$-dimensional manifold $M_4 \times G$ where G is a D-dimensional Lie group which defines the internal interaction. A defect on this manifold will be thought of as being generated in a generalized Volterra process in the following way: The manifold $\mathcal{M} \times G$ with the Minkowski space $\mathcal{M}$ is cut along a hypersurface. One of the cut faces is displaced by a transformation of $\mathcal{M} \times G$ and the hypersurfaces obtained are identified where possibly space must be added or removed. The resulting defect represents the boundary of the hypersurface and is of dimension $4 + D - 2$. We limit ourselves to defect topologies that are 2-dimensional in space-time.

The model employs a particular transformation group of $\mathcal{M} \times G$ which we denote by $P^\infty G$. This group consists of Poincaré transformations of $\mathcal{M}$ as well as internal G-transformations that are functions on $\mathcal{M}$. The vector fields generating $P^\infty G$ are given by

$$P_a = \partial_a, \qquad J_{ab} = x_a \partial_b - x_b \partial_a, \qquad S_\alpha^{(k)a\cdots d} = \underbrace{x^a \cdots x^d}_{k-\text{times}} v_\alpha, \quad k = 0, 1, 2, \ldots \quad , \quad (8)$$

where x^a are Cartesian coordinates on $\mathcal{M}$ and v_α ($\alpha = 1, \ldots, D$) are the generators of G (a basis of left invariant vector fields on G). The vector fields (8) satisfy the following commutation relations:

$$[J_{ab}, J_{cd}] = 2\eta_{a[c} J_{d]b} - 2\eta_{b[c} J_{d]a}, \qquad [J_{ab}, P_c] = 2\eta_{c[b} P_{a]}, \qquad [P_a, P_b] = 0, \qquad (9)$$

$$[J_{ab}, S_\alpha^{(k)cd\cdots f}] = 2k\, \delta_{[a}^{(c} S^{(k)}{}_{b]\alpha}^{d\cdots f)}, \qquad [P_a, S_\alpha^{(k)bc\cdots f}] = k\, \delta_a^{(b} S_\alpha^{(k-1)c\cdots f)}, \qquad (10)$$

$$[S_\alpha^{(k)a\cdots c}, S_\beta^{(l)d\cdots f}] = c_{\alpha\beta}^\gamma S_\gamma^{(k+l)a\cdots cd\cdots f}, \quad k, l = 0, 1, 2, \ldots \quad . \quad (11)$$

Here, $c_{\alpha\beta}^\gamma$ are the structure constants of G and round brackets denote symmetrization. The first three commutators form the Poincaré algebra. In the special case that G is Abelian, the commutators (11) vanish and we can define the finite dimensional group $P^n G$ which is generated by the generators (8) where $S_\alpha^{(k)a\cdots d} = 0$ for $k > n$. If we omit the generators P_a and $S_\alpha^{(0)}$ from (8), the remaining vector fields

generate the subgroup $P_0^\infty G$ of $P^\infty G$, or the subgroup $P_0^n G$ of $P^n G$ if G is Abelian. Since the generators $S_\alpha^{(0)}$ are constant on $\mathcal{M}$, we treat them on the same footing as the translations P_a.

Our aim is to describe defects within the framework of differential geometry. If on a manifold there is given a globally flat connection Γ_0 with respect to a transformation group H as structure group, the Volterra process gives rise to a locally flat connection Γ as long as the transformation in the Volterra process is in H. The defect geometry is characterized by nontrivial holonomies of Γ. In the case at hand, we therefore seek locally flat connections with the group $P^\infty G$ as structure group. We will follow the procedure of gauge theories of gravitation in that we consider Cartan connections [16]. These arise from $P^\infty G$ connections through a symmetry breaking $P^\infty G \to P_0^\infty G$ and have the advantage that the translational part can be related to a basis of cotangent space. Assume that on a principle fibre bundle P over $M_4 \times G$ with structure group $P_0^\infty G$ a Cartan connection with connection form ω taking values in the Lie algebra of $P^\infty G$ is given. By means of a section s of P we can define a gauge connection 1-form $A = s^*\omega$ on $M_4 \times G$ as the pull-back of ω. We can decompose A in the following way:

$$A = e^a P_a + \sigma^{(0)\alpha} S_\alpha^{(0)} + \frac{1}{2}\omega^{ab} J_{ab} + \sigma_a^{(1)\alpha} S_\alpha^{(1)a} + \sigma_{ab}^{(2)\alpha} S_\alpha^{(2)ab} + \cdots \quad , \qquad (12)$$

where the 1-forms e^a and $\sigma^{(0)\alpha}$ are a basis of the cotangent space at each point of $M_4 \times G$. The field strength $F = dA + A \wedge A$ is written as

$$F = T^a P_a + K^{(0)\alpha} S_\alpha^{(0)} + \frac{1}{2}R^{ab} J_{ab} + K_a^{(1)\alpha} S_\alpha^{(1)a} + K_{ab}^{(2)\alpha} S_\alpha^{(2)ab} + \cdots . \qquad (13)$$

Here, T^a and $K^{(0)\alpha}$ are torsion tensors, R^{ab} and $K_a^{(1)\alpha}$ curvature tensors, and $K_{a\cdots d}^{(k)\alpha}$ for $k > 1$ will be referred to as curvature tensors of higher order.

Locally flat defect connections are characterized by $F = 0$ on the manifold $M_4 \times G \setminus \Sigma$ where Σ is the subspace of the defect. These field equations read in components:

$$T^a = de^a + \omega^a{}_b \wedge e^b = 0, \qquad R^{ab} = d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb} = 0, \qquad (14)$$

$$K_{ab\cdots de\cdots g}^{(k)\alpha} = d\sigma_{a\cdots g}^{(k)\alpha} + \frac{1}{2}c_{\beta\gamma}^\alpha \sum_{l=0}^{k} \sigma_{(a\cdots d}^{(l)\beta} \wedge \sigma_{e\cdots g)}^{(k-l)\gamma}$$

$$- k\,\omega^h{}_{(a} \wedge \sigma_{b\cdots g)h}^{(k)\alpha} - (k+1)\,\sigma_{a\cdots gh}^{(k+1)\alpha} \wedge e^h = 0, \quad k = 0, 1, 2, \ldots \ (15)$$

Given a solution to these equations, we can compute the holonomy

$$\Lambda(*, C) = \mathcal{P}\exp\left(-\oint_C A\right)$$

along a closed curve C in $M_4 \times G \setminus \Sigma$ with base point $*$. Λ is invariant under deformations of C as long as the base point is held fixed.

We are now in a position to formulate the model for the classification of topological quantum phases: Given a 1-parameter subgroup of the group $P^\infty G$ generated by

the vector field v on $M_4 \times G$, we associate to it a quantum mechanical operator $\hat{v} = i\hbar v$ which is interpreted as the charge operator of the quantum mechanical system that encircles the "flux line" in the interference experiment. In the Volterra process, the 1-parameter subgroup generates a defect the gauge field of which can be determined with the help of the field equations (14,15) where F has a v-valued singularity concentrated on Σ. The holonomy of the gauge connection is interpreted as a phase operator acting on the wave function of the quantum mechanical system. We require that the wave function is an eigenfunction of the operator $\hat{v}$. The phase operator then becomes a phase factor which gives the topological quantum phase.

With each of the generators (8) there is associated a charge. The hierarchy of the generators $S_\alpha^{(k)a...d}$ corresponds to the hierarchy of multipole moments of the charge $S_\alpha^{(0)}$. Given a topological quantum phase, we can find the 1-parameter group that characterizes the quantum mechanical system that interferes as well as the "flux line" which represents a topological defect where the group parameter gives its strength. On the other hand, choosing a 1-parameter subgroup of $P^\infty G$ with a given G a new quantum phase can be determined. In this case, it is, however, not ensured that this phase is realized in nature since the model is purely topological and does not take into account the real interactions. For example, whether the quantum mechanical systems experience classical forces cannot be predicted from the model.

We will give a few examples to the model:

(1) Let G be the trivial group I. In this case, $P^\infty I$ is the Poincaré group and the field equations reduce to the equations (14). The defects that can be generated by means of Poincaré transformations in the Volterra process are space-time dislocations and disclinations as explained in section 2. The associated charges are mass and spin, leading to the gravitational AB and AC effect, respectively.

(2) We consider $G = U(1) \times U(1)$ corresponding to electromagnetism with magnetic and electric flux. We limit ourselves to the group $P^2(U(1) \times U(1))$ and require further that the Lorentz connection is flat and torsionfree choosing $\omega^a{}_b = 0, e^a = dx^a$. The field equations (14,15) then reduce to

$$K^{(0)\alpha} = d\sigma^{(0)\alpha} - \sigma_a^{(1)\alpha} \wedge dx^a = 0,$$

$$K_a^{(1)\alpha} = d\sigma_a^{(1)\alpha} - 2\sigma_{ab}^{(2)\alpha} \wedge dx^b = 0,$$

$$K_{ab}^{(2)\alpha} = d\sigma_{ab}^{(2)\alpha} = 0, \qquad \alpha = 1, 2. \tag{16}$$

(2a) In the case of the AB effect the quantum mechanical system is an electrically or magnetically charged particle. Thus, the associated 1-parameter subgroup of $P^2(U(1) \times U(1))$ consists of internal $U(1)$ transformations constant on $\mathcal{M}$. Only the torsion $K^{(0)\alpha}$ is nonvanishing and singular on the flux line where $\alpha = 1$ corresponds to magnetic flux and $\alpha = 2$ to electric flux. These flux lines are screw dislocations on the space $\mathcal{M} \times S^1 \times S^1$. Choosing $\sigma_a^{(1)\alpha} = \sigma_{ab}^{(2)\alpha} = 0$ the holonomy of the connection $\sigma^{(0)\alpha}$, which represents the magnetic ($\alpha = 1$) or electric ($\alpha = 2$) 4-vector potential, gives the AB phase factor.

(2b) For the AC effect the interfering system is a dipole moment. The associated 1-parameter subgroups of $P^2(U(1) \times U(1))$ are $U(1)$ transformations which are linear

on $\mathcal{M}$. The corresponding defects are disclinations characterized by singularities of the curvature $K_a^{(1)\alpha}$ with $\alpha = 1$ for electric dipoles and $\alpha = 2$ for magnetic ones. The field equations (16) are solved by

$$\sigma_{ab}^{(2)\alpha} = 0, \qquad \sigma_a^{(1)\alpha} = \frac{k_a^\alpha}{2\pi} d\varphi, \qquad \sigma^{(0)\alpha} = d\theta^\alpha - \frac{k_a^\alpha}{2\pi} x^a d\varphi, \qquad (17)$$

where the defect lies along the z-axis and φ is the azimuthal angle. k_a^α is the defect (or group) parameter and θ^α are (angular) coordinates on $S^1 \times S^1$. In the case that only k_3^α is nonvanishing, the holonomy of the connection (17) gives the AC phase factor. Alternatively, we can use a teleparallel formulation setting $K_a^{(1)\alpha} = 0$. Then, with $\sigma_a^{(1)\alpha} = 0$, $\sigma^{(0)\alpha}$ in equation (17) gives a nonvanishing torsion $K^{(0)\alpha}$ which is the field strength ($\alpha = 1$) or the dual field strength ($\alpha = 2$). If only k_3^α is nonvanishing, $K^{(0)1}$ is the field strength of a homogeneous magnetic line charge and $K^{(0)2}$ that of an electric one.

(2c) In the case that the curvature of second order $K_{ab}^{(2)\alpha}$ is singular on the defect, we obtain a topological quantum phases for quadrupole moments. The "flux line" corresponds to a defect which results in the Volterra process from a $U(1)$ transformation quadratic on $\mathcal{M}$. Again, a teleparallel formulation is possible where we have a nonvanishing $K^{(0)\alpha}$.

4   Conclusion

In this letter, we have proposed a model that allows a classification of topological quantum phases in that an element of a transformation group of a higher dimensional space-time is associated to each quantum phase. Our starting point was the principle that topological quantum phases arise in the scattering from space-time defects. The phase factors are then given by the holonomies of the defect geometries. The model provides moreover an explanation why some quantum phases in electromagnetism are usually not described by holonomies of locally flat connections: From the viewpoint of a higher dimensional unification, Maxwell's theory possesses the nature of a teleparallel theory.

We close with a few remarks:

(1) The gauge fields of higher order we have introduced do not seem to be suitable for a formulation of a dynamics of gauge fields in the general case. They are only used here to show that the "flux lines" in the topological quantum phases represent space-time defects. Kaluza-Klein theory gives a formulation of electromagnetism in terms of linear connections, the corresponding symmetries are, however, broken.

(2) A crucial property of the model is the combination of external and internal transformations in the gauge group. As a result, the field equations (14,15) of different order are coupled. This has the consequence that a defect described by a curvature of a given order can be represented as a pair (dipole) of defects of one order higher.

(3) At least in the case that G is Abelian, there is a close relation between principle $P^\infty G$-bundles and bundles of frames of higher order over $M_4 \times G$ [17].

(4) There exists another attempt to formulate the electromagnetic AC effect by means of a holonomy of a connection [18][19] using the fact that a neutral particle with magnetic moment in an electromagnetic field is equivalent to an isospin particle in an $SU(2)$ gauge field [20][21]. However, the $SU(2)$ connection is not flat and cannot be interpreted as describing a topological defect. Moreover, the $SU(2)$ symmetry originates from the spin of the particle.

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